

$\frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{6}}$ の分母を有理化せよ.

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解説

$$\begin{aligned}(\text{与式}) &= \frac{\sqrt{2} + \sqrt{3} - \sqrt{6}}{\{(\sqrt{2} + \sqrt{3}) + \sqrt{6}\} \{(\sqrt{2} + \sqrt{3}) - \sqrt{6}\}} \\&= \frac{\sqrt{2} + \sqrt{3} - \sqrt{6}}{(\sqrt{2} + \sqrt{3})^2 - (\sqrt{6})^2} \\&= \frac{\sqrt{2} + \sqrt{3} - \sqrt{6}}{2\sqrt{6} - 1} \\&= \frac{(\sqrt{2} + \sqrt{3} - \sqrt{6})(2\sqrt{6} + 1)}{(2\sqrt{6} - 1)(2\sqrt{6} + 1)} \\&= \frac{4\sqrt{3} + \sqrt{2} + 6\sqrt{2} + \sqrt{3} - 12 - \sqrt{6}}{23} \\&= \frac{-12 + 7\sqrt{2} + 5\sqrt{3} - \sqrt{6}}{23}\end{aligned}$$