

$\frac{1}{1+\sqrt{2}+\sqrt{3}+\sqrt{6}}$ の分母を有理化せよ。

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解説

$$\begin{aligned}\frac{1}{1+\sqrt{2}+\sqrt{3}+\sqrt{6}} &= \frac{1}{(\sqrt{6}+1)+(\sqrt{3}+\sqrt{2})} \cdot \frac{(\sqrt{6}+1)-(\sqrt{3}+\sqrt{2})}{(\sqrt{6}+1)-(\sqrt{3}+\sqrt{2})} \\ &= \frac{(\sqrt{6}+1)-(\sqrt{3}+\sqrt{2})}{(\sqrt{6}+1)^2-(\sqrt{3}+\sqrt{2})^2} \\ &= \frac{1-\sqrt{2}-\sqrt{3}+\sqrt{6}}{7+2\sqrt{6}-(5+2\sqrt{6})} \\ &= \frac{1-\sqrt{2}-\sqrt{3}+\sqrt{6}}{2}\end{aligned}$$

別解

$$\begin{aligned}\frac{1}{1+\sqrt{2}+\sqrt{3}+\sqrt{6}} &= \frac{1}{(1+\sqrt{2})+\sqrt{3}(1+\sqrt{2})} \\ &= \frac{1}{(1+\sqrt{2})(1+\sqrt{3})} \\ &= \frac{1}{(\sqrt{2}+1)(\sqrt{3}+1)} \cdot \frac{(\sqrt{2}-1)(\sqrt{3}-1)}{(\sqrt{2}-1)(\sqrt{3}-1)} \\ &= \frac{1-\sqrt{2}-\sqrt{3}+\sqrt{6}}{2}\end{aligned}$$