(1)　等式$\displaystyle\int_{-1}^{0}\frac{x^2}{1+e^x}dx=\int_0^1\frac{x^2}{1+e^{-x}}dx$ を示せ。

(2)　定積分$\displaystyle\int_{-1}^{1}\frac{x^2}{1+e^x}dx$ を求めよ。

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(解説)

(1) $\displaystyle\int_{-1}^{0}\frac{x^2}{1+e^x}dx$

x	$-1\longrightarrow 0$
t	$1\longrightarrow 0$

$x=-t$ とおくと，$\dfrac{dx}{dt}=-1$

$$\int_{-1}^{0}\frac{x^2}{1+e^x}dx=\int_1^0\frac{(-t)^2}{1+e^{-t}}(-dt)=\int_0^1\frac{t^2}{1+e^{-t}}dt=\int_0^1\frac{x^2}{1+e^{-x}}dx$$

(2)
$$\begin{aligned}\int_{-1}^{1}\frac{x^2}{1+e^x}dx&=\int_{-1}^{0}\frac{x^2}{1+e^x}dx+\int_0^1\frac{x^2}{1+e^x}dx\\&=\int_0^1\frac{x^2}{1+e^{-x}}dx+\int_0^1\frac{x^2}{1+e^x}dx\\&=\int_0^1\left(\frac{x^2e^x}{1+e^x}+\frac{x^2}{1+e^x}\right)dx\\&=\int_0^1\frac{x^2(1+e^x)}{1+e^x}dx\\&=\int_0^1x^2dx=\left[\frac{1}{3}x^3\right]_0^1=\frac{1}{3}\end{aligned}$$